Uncertainty Quantification on Parameters of the Soil-Carbon System

Eric Cawi
Mentor: Dr. Sauer

Dept. of Mathematical Sciences
George Mason University

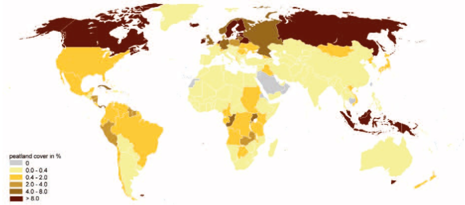
October 11, 2013

Objectives

- Describe the dynamics of soil temperature and CO_2 content in a carbon system
- Identify instabilities and dangers in the system
- Use two distinct approaches to analyze the sensitivity of the unstable regions

Background: Peat Deposits

- Peat: soil characterized by high contents of partially decayed vegetation
- Roughly 1/3 of the world's CO_2 is trapped in global peat deposits
- Peat is used as a fuel source in developing countries, and can also spontaneously ignite under certain conditions



Background: Soil temperature and CO_2 content in a peat deposit

The rate of change of soil temperature (T(t)) and carbon content (C(t)) are given by the following equations(Wieczorek et al, 2010):

$$C'(t) = \Pi - r_0 C(t) e^{\alpha T(t)}$$

$$T'(t) = \frac{1}{\epsilon}(r_0 C(t) e^{\alpha T(t)} - \frac{\lambda}{A}(T(t) - T_a))$$

where Π,r_0, α, ϵ, A, and T_a are physical parameters of the system such as rate of decaying vegetation (Π) and atmospheric temperature (T_a) .

Modifications to the Soil - Carbon system

In order to simplify the problem, two modifications were made to the system:

- Substitute U(t) = T(t) - Teq to center the soil temperature around the equilibrium point

- Use a Taylor expansion to substitute $e^{\alpha T_{eq}}(1 + \alpha U(t) + ...)$ for the $e^{\alpha(U(t)+T_{eq})}$ term.

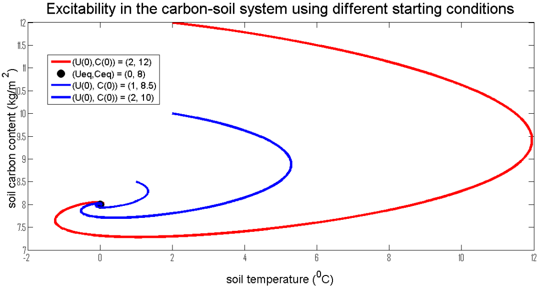
So the modified system is:

$$C'(t) = \Pi - r_0 C(t) e^{\alpha T_{eq}}(1 + \alpha U(t))$$

$$U'(t) = \frac{1}{\epsilon}(r_0 C(t) e^{\alpha T_{eq}}(1 + \alpha U(t)) - \frac{\lambda \Pi}{A}(U(t) + T_{eq} - T_a))$$

Solutions and Excitability in the System

While the solution always travels toward an equilibrium point at $(T_{eq}, C_{eq}) = (T_a + \frac{\Pi A}{\lambda}, \Pi r_0(1 + \alpha T_{eq}))$, the path of the solution depends on the initial state of the soil.



When the system starts with too much carbon, it can become excited, leading to a dramatic rise and fall in soil temperature

The Danger Zone

Physically, a peat deposit may never recover from such an increase in soil temperature, and ignition becomes an increasingly potent danger. If the excitability threshold around the equilibrium is crossed in other ways, catastrophic releases of CO_2 can occur in addition to the rise in temperature.

the excitability threshold for this region is called the "compost bomb instability"

Uncertainty Quantification

This research examines the sensativity of the physical parameters in this system. For example, no two peat deposits have the same amount of vegetation decay, but changes in Π will change the size of the excitability threshold. To measure the excitability caused by changes in vegetation decay, the value of Π was set to $\Pi_0 + \sigma z$, where z was taken from a normal distribution centered around the original value of Π with standard deviation σ. Therefore,

$$C'(t) = \Pi_0 + \sigma z - r_0 C(t) e^{\alpha T_{eq}}(1 + \alpha U(t))$$

is the new equation governing the soil carbon content. The new system is solved for many different values of z, and measuring the corresponding soil temperature gives a measure of excitability.

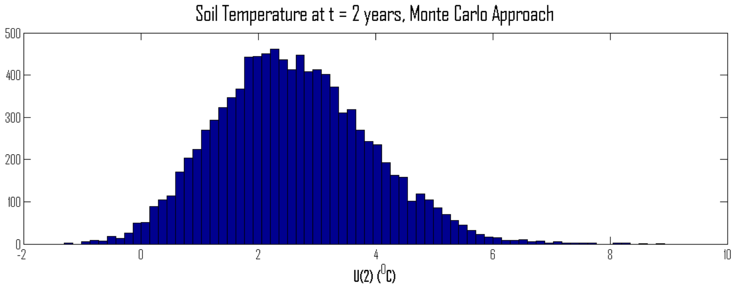
Approach One: Monte Carlo

The first approach used to measure the excitability in the soil temperature is known as a Monte Carlo Simulation (named after the casino):

- Set a value of z, and solve the system of equations up until t = 2 years
- Record the soil temperature at t = 2 years
- Repeat for 10,000 different values of z

The advantage of the Monte Carlo approach is in its brute force simplicity. As long as the equation has a decent solution, Monte Carlo will output a histogram of 10,000 different ending temperatures at any desired accuracy.

Results: Monte Carlo



From this histogram, most of the ending temperatures are not excited. However, a good portion rise above $4^\circ C$ above the equilibrium temperature. This occurs when the vegetation decay rate is much higher than it's original value.

Appoach Two: Generalized Polynomial Chaos

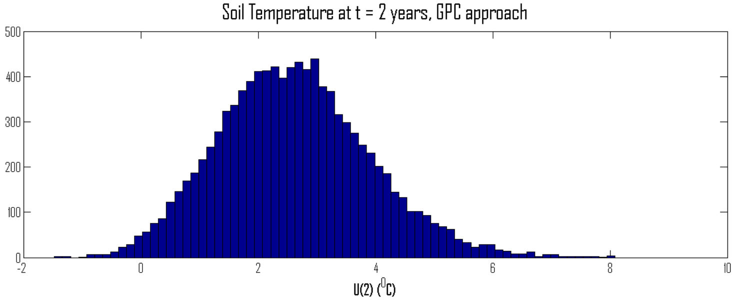
Another approach uses a special set of orthogonal polynomials to "factor" out the random value z. This method changes hte problem to a set of 2N+2 equations where the kth equation is given by :

$$k!C_k'(t) = \{^{\Pi k=0}_{0 else} + \{^{\sigma k=1}_{0 else} - r_0 e^{\alpha T_{eq}}(k!C_k(t) + \sum_{i=0}^{n} C_i(t) * \sum_{i=0}^{n} U_i(t) * e_{ijk})$$

$$k!U_k'(t) = r_0 e^{\alpha T_{eq}}(k!C_k(t) + \sum_{i=0}^{n} C_i(t) * \sum_{i=0}^{n} U_i(t) * e_{ijk}) - \frac{\lambda \Pi}{A}(U(t) + \{^{T_{eq} - T_a k=0}_{0 else})$$

This system is only solved once, and then evaluated at 10,000 different values of z.

Results: GPC



The resulting histogram from 10,000 different values of z is nearly identical to the Monte Carlo approach, as expected.

Costs and Benefits of GPC

Advantages:

- GPC is more efficient, because only one system of equations is solved. For 10,000 values of Π, the GPC was 10 times faster than the Monte Carlo simulation.
- For different probability distributions, different sets of orthogonal polynomials can simply be substituted into the equations.

Disadvantages:

- GPC is far more complex than Monte Carlo, and a solution to the system of equations is not always apparent.
- When multiple parameters are varied simultaneously, GPC becomes less efficient computationally than Monte Carlo

The Big Picture

- ClImatology systems are often characterized by bifurcations or tipping points similar to those seen in a peat deposit.
- Identifying tipping points is critical in understanding the climate, as one catastrophe can cause a cascade of other disasters.
- When a tipping point has been identified, scientists can attempt to control or diminish the danger.

Future Work

- Run Monte Carlo simulations for simultaneously varied parameters, hopefully see the emergence of a "stability region"
- Improve the exponential Taylor Expansion to second order accuracy
- Instead of using a normal distribution, turn the system into a set of partial differential equations relating space (possibly represented by latitude and longitude) and time to soil and carbon content. The simplest case would be to examine different peat deposits on a grid, and the previously constant parameters would become functions depending on x and y.

References

- GJoosten, H. (2010, August). The global peatland co2 picture. Retrieved from http://www.wetlands.org/WatchRead/Currentpublications/tabid/56/ Global-Peatland-CO2-Picture.aspx
- Lindsay, R. (2010, June). Peat and carbon report. Retrieved from http://www.uel.ac.uk/erg/PeatandCarbonReport.htm
- Wieczorek, S., Ashwin, P., Luke, C., and Cox, P. (2010). Excitability in ramped systems: the compost-bomb instability. Proceedings of the Royal Society, published online, doi: 10.1098
- Xiu, D. (2010). Numerical methods for stochastic differential equations: a spectral method approach. (pp. 68-70). Princeton, New Jersey: Princeton University Press.