Paleoclimatology and Climate Field Reconstruction

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Abstract

In climate change research, it is important to utilize accurate historical temperature approximations. Most methods use proxies such as tree rings to develop their approximations. We study these methods to develop error bounds for the estimates and discover the effect of adding other explanatory variables like carbon dioxide.

The Singular Value Decomposition

The singular value decomposition(SVD) of a matrix A separates a matrix into the product of three different matrices, U, S, and V^T

In this research: A= matrix of temperature anomalies, U = time series from the temperature matrix, V^T = spatial factors, and S = influence of the time series on the spatial factors.

$$\begin{pmatrix} A_{11} & \cdot\cdot & A_{1n} \\ \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot\cdot & \cdot \\ A_{m1} & \cdot\cdot & A_{mn} \end{pmatrix} = \begin{pmatrix} U_{11} & \cdot\cdot & U_{1m} \\ \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot\cdot & \cdot \\ U_{m1} & \cdot\cdot & U_{mm} \end{pmatrix} * \begin{pmatrix} S_{11} & \cdots & 0 \\ 0 & S_{22} 0\cdot & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & S_{nn} \\ \cdot & \cdots & 0 \\ \cdot & \cdots & \cdot \\ 0 & \cdots & 0 \end{pmatrix} * \begin{pmatrix} V_{11}^T & \cdot\cdot & V_{1n}^T \\ \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot\cdot & \cdot \\ V_{n1}^T & \cdot\cdot & V_{nn}^T \end{pmatrix}$$

Mann's Reconstruction: 1998

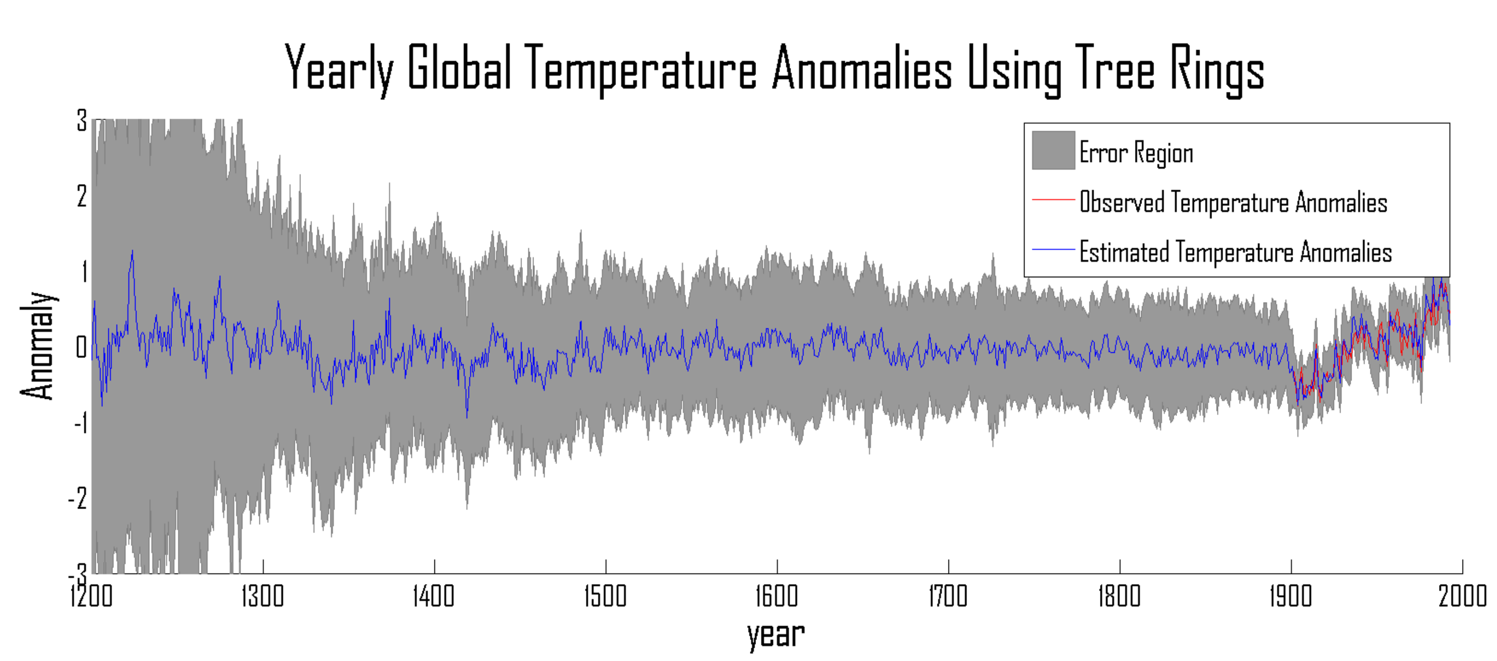
Original Equation Solves for a relationship G between the time series U and tree ring growth indeces P:

$$U_{modern}G^T = P_{modern}$$

The historical time series is found by "transposing" the original equation:

$$GU_{past}^T = P_{past}^T$$

Using the SVD the reconstructed temperature matrix is $A_{past} = U_{past}SV^T$



Improving Mann's Method

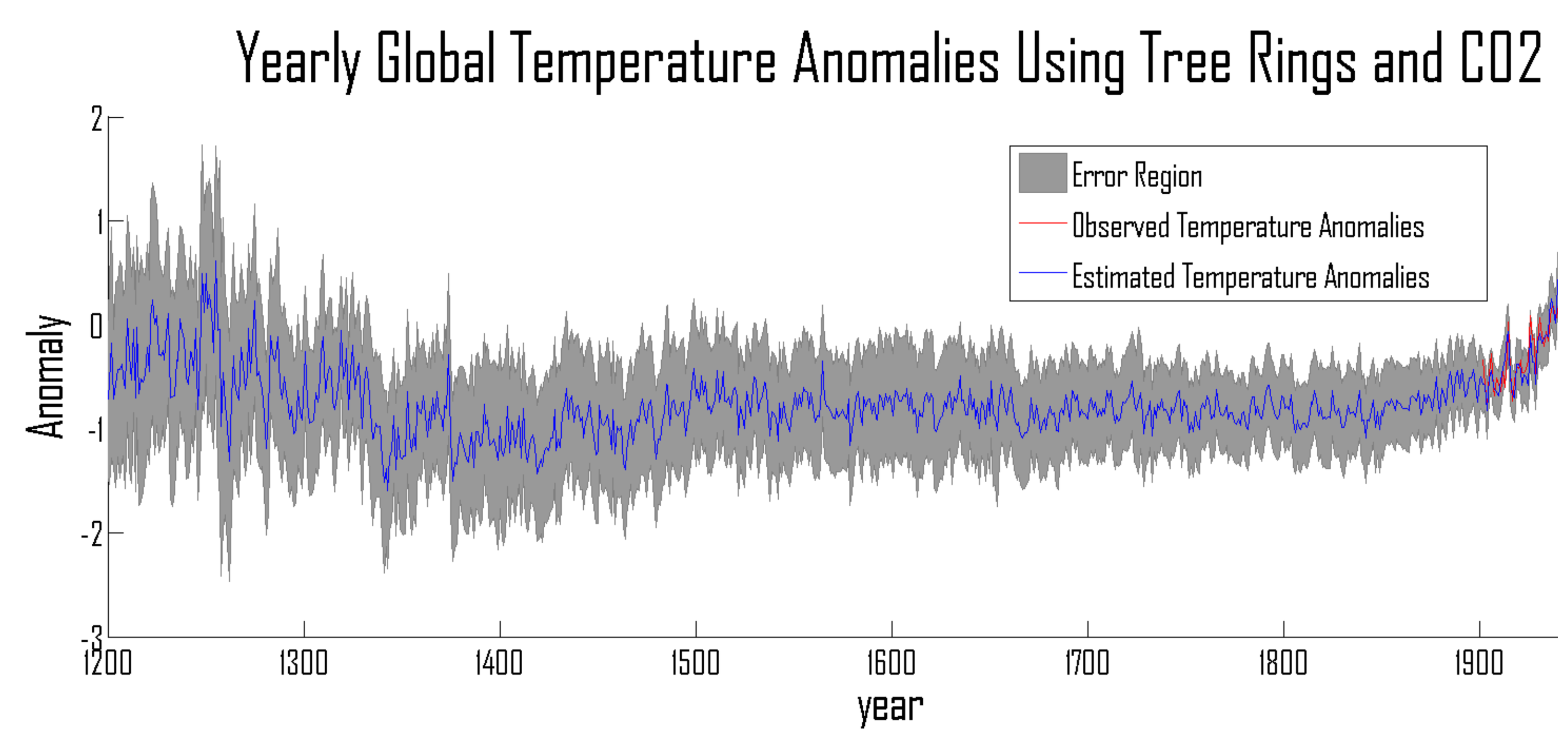
It can be assumed that temperatures are not the only thing affecting tree growth. We add a component for CO_2, C, and a noise constant, B:

$$U_{modern}G^T + CH + B = P_{modern}$$

When this equation is solved, the time series can be isolated:

$$GU_{past}^T = P_{past}^T - CH - B$$

The temperature matrix is then reconstructed as before



Calculating the Error Bars

We want to calculate the error in the temperature approximations, $A - A_{past} = (U - U_{past})SV^T$

Assuming our model is correct, we have that

$$E_p = P - P_{past} = GU - GU_{past}$$

However, since the model is a least squares solution, we can utilize the normal equations by multiplying through by G^T

$$G^T E_P = G^T G(U - U_{past})$$

or

$$U - U_{past} = (G^T G)^{-1} G^T E_P$$

Taking the norm of both sides gives

$$\|U - U_2\| \leq \|(G^T G)^{-1}\| \|G^T\| \|E_p\|$$

So

$$\|A - A_2\| \leq \|U - U_2\| * \|S\| * \|V^T\| \leq \|(G^T G)^{-1}\| * \|G^T\| * \|E_p\| * S(1,1)$$

Using the formula for standard error with M = number of sites,

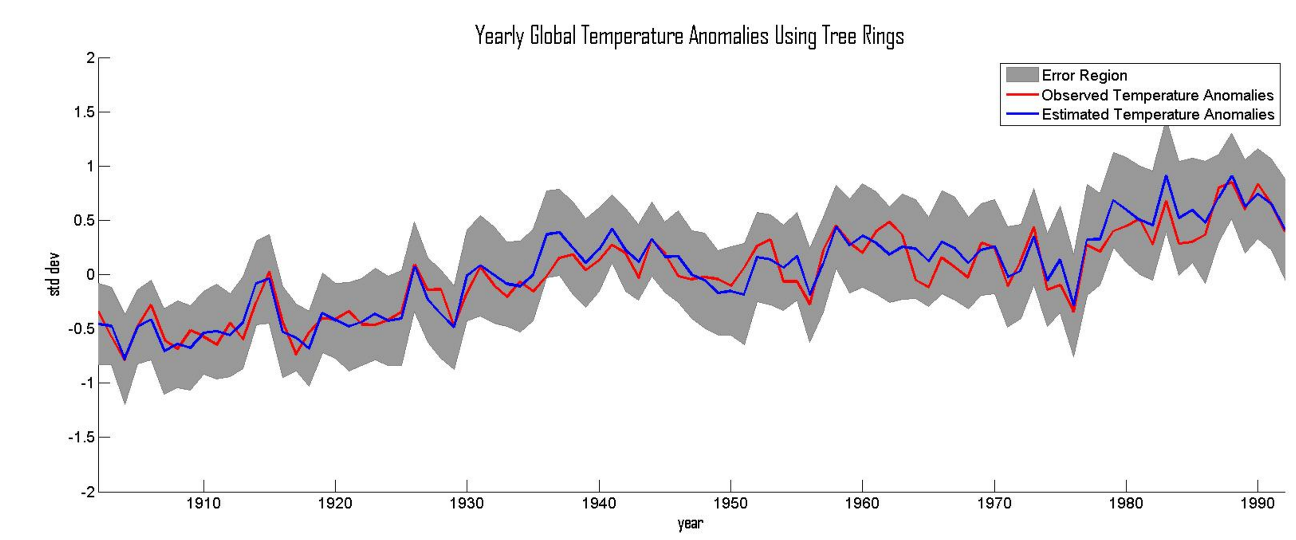
$$E_A \leq \frac{\|A - A_2\|}{M}$$

Validating the Models

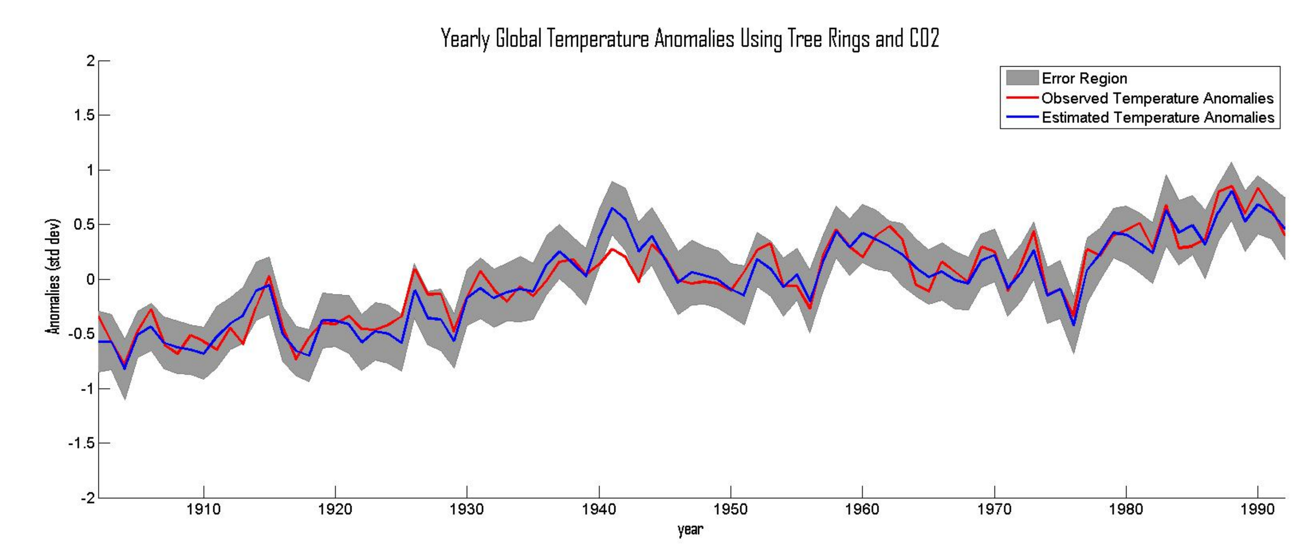
To examine the impact of CO_2, the model was crossvalidated by:

- Remove 32 random years from the training period
- Run both models to predict the omitted years
- Calculate the Root Mean Square Error for each model

Without CO2: RMSE = 0.238



With CO2: RMSE =0 .177



The addition of CO_2 provides a clear improvement on the reconstruction, especially when approximating far back into the past.

References

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