Uncertainty Quantification for the Soil-Carbon System using Monte Carlo and GPC

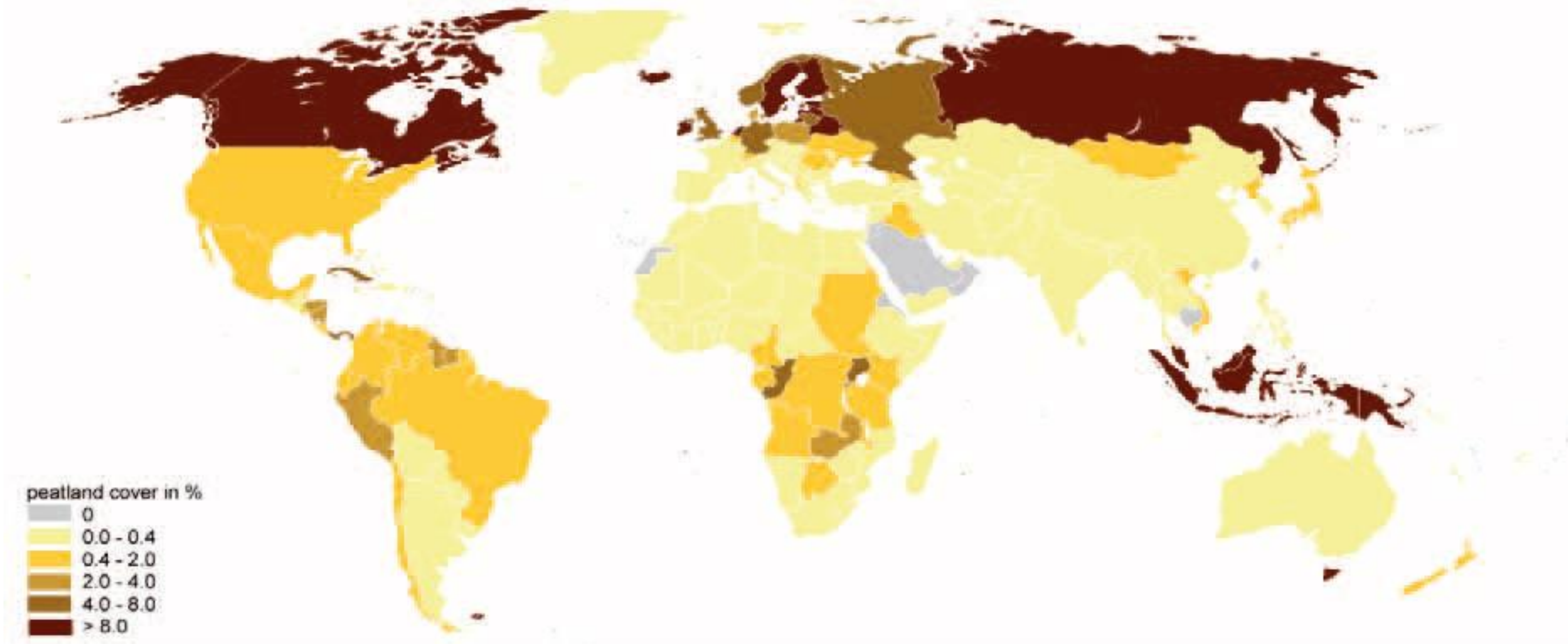
Eric Cawi, ecawi@gmu.edu

Department of Mathematical Sciences, George Mason University

Introduction

The goal of this research is to determine how changing different physical parameters in a peat deposit affects the temperature and carbon content of the soil.

Scientists estimate that one third of the world's carbon is stored in peat deposits across the globe. However, several threats to these peat reserves exist. Many countries burn peat for an important source of fuel. This releases carbon into the atmosphere and severely damages the peat deposit ecosystem, which can then take centuries to recover the previous store of carbon. Deforestation and wetland draining have also had a substantial impact on peat reserves, further releasing carbon into the atmosphere. An estimated 6% of the world's carbon emissions, in excess of 2 Gigatons is the result of human destruction of peat deposits. This estimate does not include carbon released by fires resulting from increased soil temperature or other natural disruptions.



Peat deposit land coverage, by country. Taken from The Global Peatland CO2 picture, wetlands.org

Excitability Threshold

The equations relating peat soil's carbon content C(t) and soil temperature T(t) (assuming a constant atmospheric temperature) are given by:

$$C'(t) = P - r_0 C(t) e^{\alpha T(t)}$$

$$T'(t) = \frac{1}{\epsilon}\left(r_0 C(t)\, e^{\alpha T(t)} - \frac{\lambda}{A}(T(t) - T_a)\right)$$

P	1.055 kg m^{-2} yr^{-1}
r0	.01 yr^{-1}
α	ln(2.5)/10 $^oC^{-1}$
ϵ	.0641 kg $^oC^{-1}$ m^{-2}
A	3.9*10^7
λ	5.049*10^6
Ta	20, ^oC

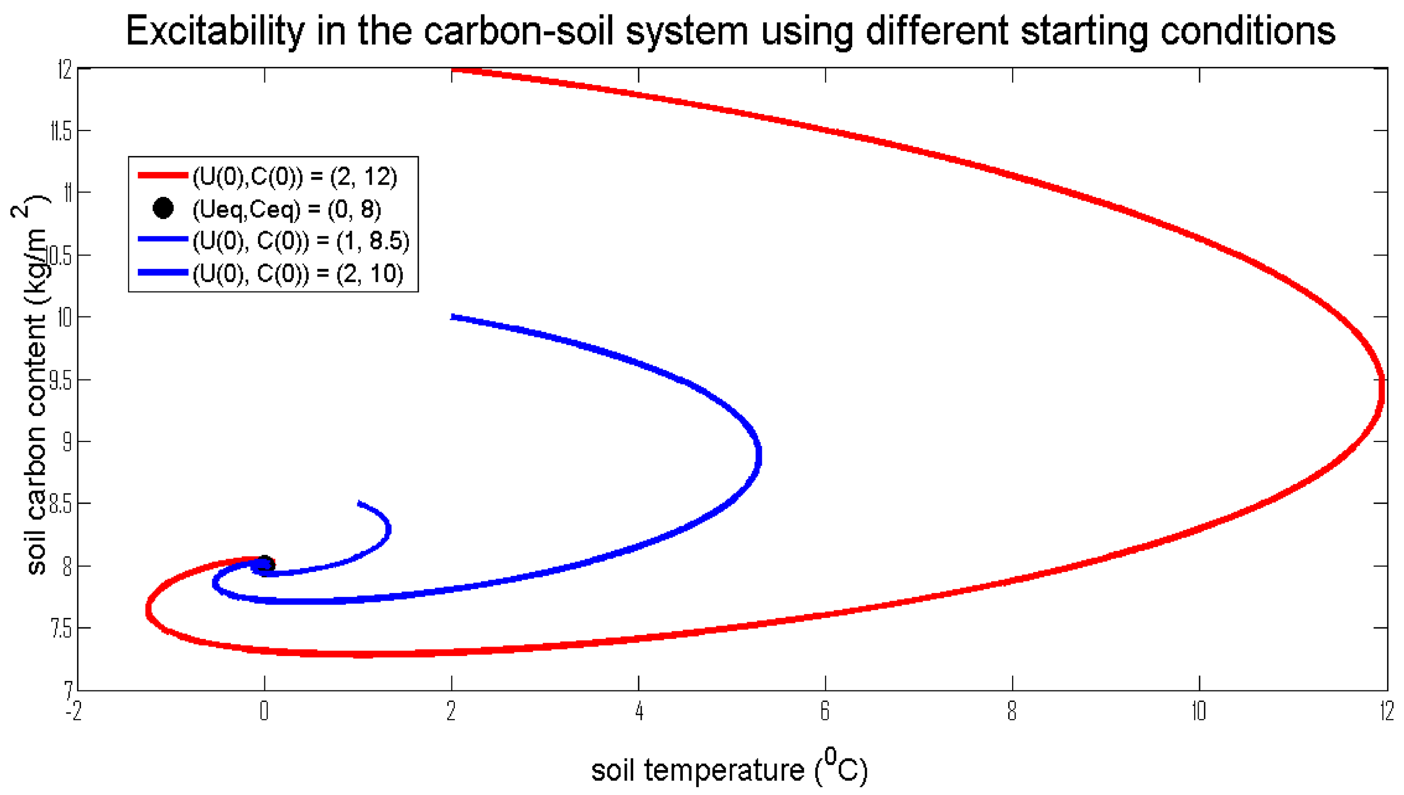
Solutions of this equation move towards the equilibrium point at $(Teq = \frac{P}{r_0 e^{\alpha T(t)}}, Ceq = T_a + \frac{AP}{\lambda})$, however, when initial conditions move past a certain threshold the solution becomes **excited**, spiraling out and taking much longer to reach the calculated equilibrium point.

Physically, this excitability represents a catastrophic increase in soil temperature. This drastically increases the risk that the carbon stored in the system will ignite and release into the atmosphere, and has been termed the "compost-bomb instability". While the model can eventually recover from spikes in temperature, peat deposits can take centuries to rebound from such a dramatic loss.

We simplified the soil carbon system in two ways. First, we defined U(t) = T(t) – Teq. This directly measures the temperature increase from the equilibrium point and simplifies many of the calculations. We also used a first order Taylor expansion in place of the $e^{\alpha T(t)} = e^{\alpha(U(t)+Teq)}$ to further simplify the calculations when using the GPC approach. The new system is:

$$C'(t) = P - r_0 e^{\alpha * Teq} C(t)\,(1 + \alpha U(t))$$

$$U'(t) = \frac{1}{\epsilon}\left(r_0 e^{\alpha * Teq} C(t)\,(1 + \alpha U(t)) - \frac{\lambda}{A}(U(t) + Teq - T_a)\right)$$



This plot shows that relatively small changes in initial conditions can lead to large excitement in the system. While the model eventually recovers from the temperature gain, the physical system might never recover.

Introducing a Random Variable: Monte Carlo Approach

The parameter P represents the carbon absorbed into the soil through decaying vegetation. Changes in P represent changes such as deforestation or conservation. To investigate changes in the parameters of the system, we used the following approach:

- Substitute P = 1.055+σ*z into the system, where z is a constant taken from a normal distribution and get a new system:

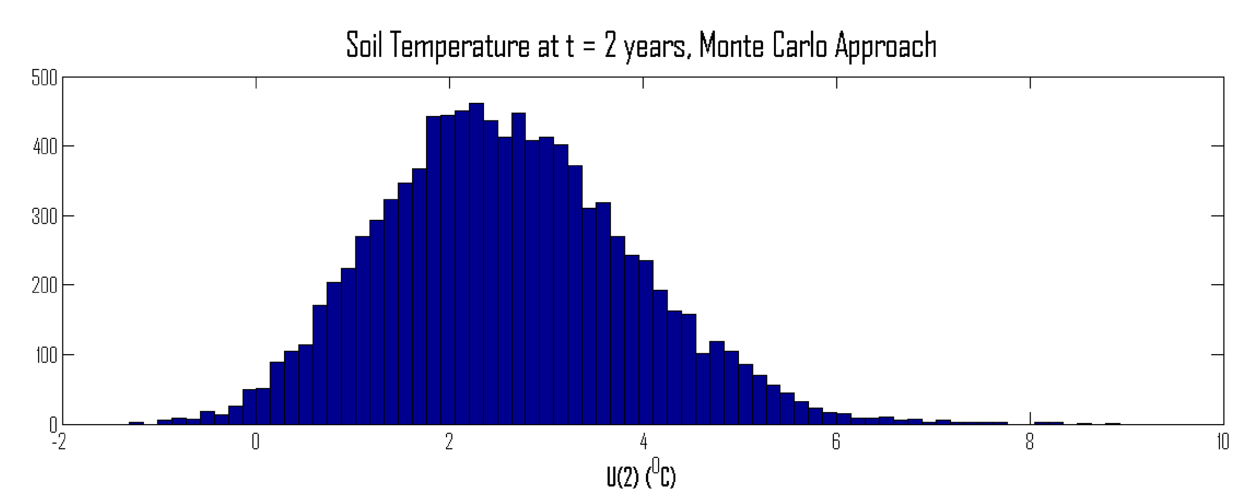
$$C'(t) = 1.055 + \sigma z - r_0 e^{\alpha * Teq} C(t)\,(1 + \alpha U(t))$$

$$U'(t) = \frac{1}{\epsilon}\left(r_0 e^{\alpha * Teq} C(t)\,(1 + \alpha U(t)) - \frac{\lambda}{A}(U(t) + Teq - T_a)\right)$$

- Using initial conditions known to cause excitability in the original system, solve the new system with a fourth order Runge Kutta Method to time t =2 and record the system's state at the ending time.
- Repeat for different values of z until desired sample size has been created. In this case we used 1000 values of z because of computation time limits.

The result of this method is a histogram detailing the excitement of the system at time t = 2 years with each of the 10,000 different z values.

RESULTS: Monte Carlo



Physically, this plot means that the system will stay less than 3^{0}C away from the final equilibrium temperature for most vegetation decay rates. However, when large amounts of carbon are input into the soil, the deposit drastically warms up, increasing the danger of ignition/carbon release. This implies that oversaturation of carbon into peat deposits can lead to rapid destabilization.

Improved Approach: Generalized Polynomial Chaos

Generalized polynomial chaos (GPC) is a different approach often used to solve equations with random variables that uses the orthogonal set of Hermite polynomials to create a single system of differential equations. For this method, we assume that the solutions $C(t) = \sum_{i=0}^{N} C_i(t) H_i(z)$ and $U(t) = \sum_{i=0}^{N} U_i(t) H_i(z)$, where Hi(z) is the i'th Hermite polynomial evaluated at the random variable z. With this substitution, the new system is:

$$C'(t) = \mu + \sigma z - r_0 e^{\alpha * Teq} \sum_{i=0}^{N} C_i(t) H_i(z)\,\left(1 + \sum_{j=0}^{N} \alpha U_j(t) H_j(z)\right)$$

$$U'(t) = \frac{1}{\epsilon}\left[r_0 e^{\alpha * Teq} \sum_{i=0}^{N} C_i(t) H_i(z)\,\left(1 + \sum_{j=0}^{N} \alpha U_j(t) H_j(z)\right) - \frac{\lambda}{A}\left(\sum_{i=0}^{N} U_i(t) H_i(z) + Teq - T_a\right)\right]$$

However, since $C'(t) = \sum_{i=0}^{N} C'_i(t) H_i(z)$ (similar for U'(t)), we can isolate and C_k'(t) using the expected value formula for any k = 0,1,2,…N. This gives us a system of 2N+2 equations of the form:

$$k!\,C'_k(t) = \begin{cases} 1.055 & k = 0 \\ 0 & else \end{cases} + \begin{cases} \sigma & k = 1 \\ 0 & else \end{cases} - r_0 e^{\alpha * Teq}\left(k!\; C_k(t) + \sum_{i=0}^{N} C_i(t) \sum_{j=0}^{N} U_j(t) e_{ijk}\right)$$

$$k!\,U'_k(t) = r_0 e^{\alpha * Teq}\left(k!\; C_k(t) + \sum_{i=0}^{N} C_i(t) \sum_{j=0}^{N} U_j(t) e_{ijk}\right) - \frac{\lambda}{A}\left(k!\,U_k(t) + \begin{cases} Teq - Ta & k = 0 \\ 0 & else \end{cases}\right)$$

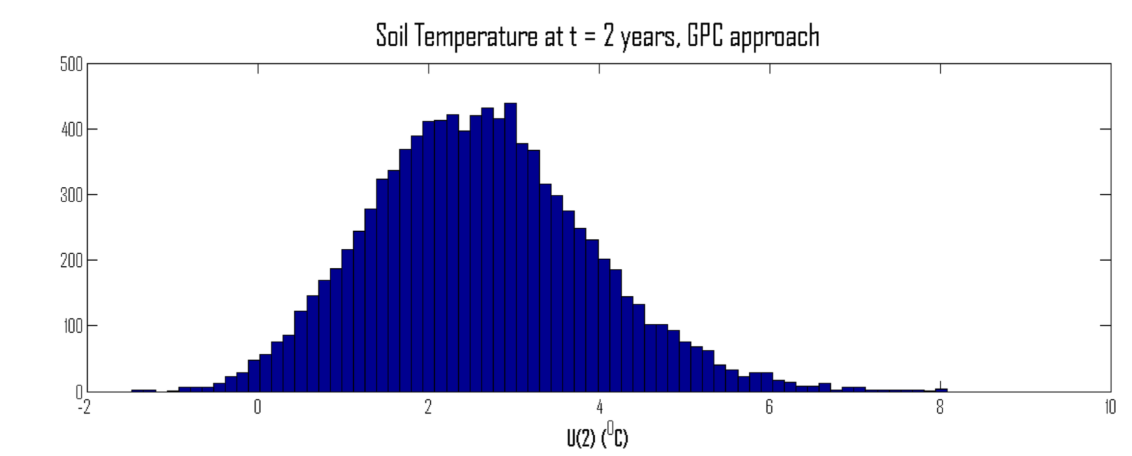
Here e_{ijk} is the expected value of $H_i(z)*H_j(z)*H_k(z)$ and

$$e_{ijk} = \begin{cases} \frac{i!j!k!}{(s-i)!(s-j)!(s-k)!} & s \geq i,j,k \text{ and } 2s = i+j+k \text{ is even} \\ 0 & else \end{cases}$$

Once this system is solved, a value of z is chosen and the sum $C(t) = \sum_{i=0}^{N} C_i(t) H_i(z)$ (similar for U(t)) is calculated for the final solution.

Results: GPC

The ending result of the GPC approach is another histogram based off of 1000 random z values.



This histogram roughly matches the histogram generated by the Monte Carlo approach, with small differences appearing because different values of z are used in every run through. The main advantage of the GPC approach is that only one system of equations is solved, rather than 10,000, so GPC runs much faster. For the histograms shown, the GPC approach took 78 seconds to execute, while the Monte Carlo approach took 912 seconds. Furthermore, if we are interested in one particular value of z, we do not have to re-solve the equations.

Currently, we are working on extending the GPC approach to a second order Taylor series expansion, which should provide a more accurate model of the soil to atmosphere temperature transfer.

Acknowledgements

Thanks to my mentor, Dr. Timothy Sauer, who has mentored me through the past 4 semesters.

This research was funded and supported by the George Mason Undergraduate research Scholars Program.

REFERENCES

GJoosten, H. (2010, August). *The global peatland co2 picture*. Retrieved from http://www.wetlands.org/WatchRead/Currentpublications/tabid/56/mod/1570/articleType/ArticleView/articleId/2418/The-Global-Peatland-CO2-Picture.aspx

Lindsay, R. (2010, June). *Peat and carbon report*. Retrieved from http://www.uel.ac.uk/erg/PeatandCarbonReport.htm

Wieczorek, S., Ashwin, P., Luke, C., & Cox, P. (2010). Excitability in ramped systems: the compost-bomb instability. *Proceedings of the Royal Society*, *published online*, doi: 10.1098

Xiu, D. (2010). *Numerical methods for stochastic differential equations: a spectral method approach*. (pp. 68-70). Princeton, New Jersey: Princeton University Press.