Paleoclimatology and Climate Field Reconstruction

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Background

- Paleoclimatology: The study of climate changes with respect to earth's entire history.
- Climate Field Reconstruction (CFR) : The use of proxies, e.g. tree rings, ice cores, coral,
  to estimate climate factors far into the past (Mann et al., 1998)

Driving Questions

- What is the best approximation for the global mean temperature in past centuries?
- What are the error bounds for this guess?
- How have environmental factors (CO_2, rainfall, solar radiation, etc.) affected global temperatures?

Current Data

Temperature Data:

- 1082 sites
- 92 Years
- In anomalies: mean of 0 and standard deviation 1 (this preserves any relationships between the numbers but makes them easier to work with)

Tree Rings

- Have some tree sites going back approx. 1000 years(collected by NOAA)
- We modified the tree data to be in anamolies as well

The Singular Value Decomposition of a Matrix

The Singular Value Decomposition is a method that splits a matrix into the product of three different matrices:

$$A = U_1 \; S \; V^T$$

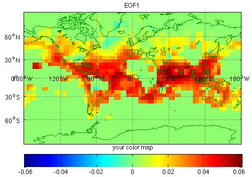
A	U_1	S	V^T
Original Matrix of Temperatures	Matrix of Time Series that shows the influence of time on the temperatures	Diagonal Matrix of singular values. Determines the influence of the columns of U and V	Matrix of spatial factors involved in temperatures

Here we are most interested in U_1 because it represents the temporal factors we wish to reconstruct back in time

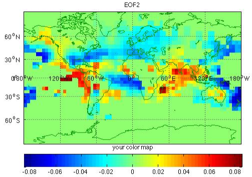
Empirical Orthogonal Functions

- The Matrix V contains the spatial relationships present in the temperature
- Each column represents part of the total climate pattern of the world.
- Only the first few columns of V are important to the data, the later columns just add noise

EOF1 - global temperature average



EOF2 - El Nino



The Time Series and Tree Rings (Mann et al, 1998)

Looking for a matrix G to relate the time series to tree ring data P_1

$$U_1 G^T = P_1^T$$

To find G, we solve the matrix:

$$\begin{pmatrix} U_{11} & \cdots & U_{1e} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ U_{N1} & \cdots & U_{Ne} \end{pmatrix} * \begin{pmatrix} G_{11} & \cdots & G_{1e} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ G_{T1} & \cdots & G_{Te} \end{pmatrix} = \begin{pmatrix} P_{1x} \\ \cdot \\ \cdot \\ \cdot \\ P_{Nx} \end{pmatrix}$$

Note: This is a least squares solution, not exact.

Reconstructing the Time Series

Now that we have G, use G and historical tree data to reconstruct the time series

by "transposing" the original equation

$$GU_2^T = P_2^T$$

G = Relation between time series and tree ring data

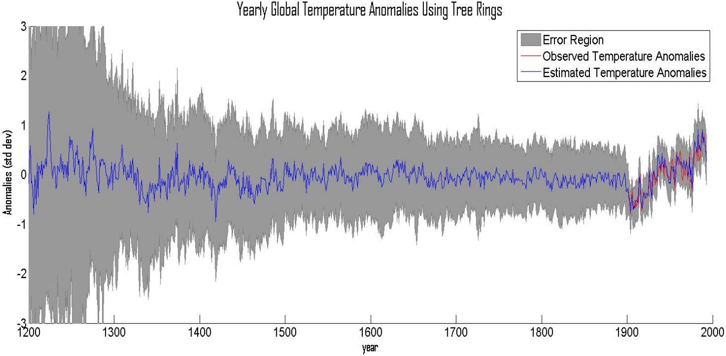
P_2 = Historical Tree Data

U_2 = Reconstructed Time series

Using U_2 to reconstruct Temperature

To reconstruct the temperatures, simply replace U_1 with U_2 in the original equation

$$A_2 = U_2 S V^T$$



Calculating The Error Bounds

We want to calculate the error in the temperature approximations, $A - A_2 = (U - U_2)SV^T$
Assuming our model is correct, we have that

$$E_p = P - P_{app} = GU - GU_2$$

However, since the model is a least squares solution, we can utilize the normal equations by multiplying through by G^T

$$G^T E_P = G^T G(U - U_2)$$

or

$$U - U_2 = (G^T G)^{-1} G^T E_P$$

Calculating The Error Bounds

Taking the norm of both sides gives

$$\|U - U_2\| \leq \|(G^T G)^{-1}\| \|G^T\| \|E_p\|$$

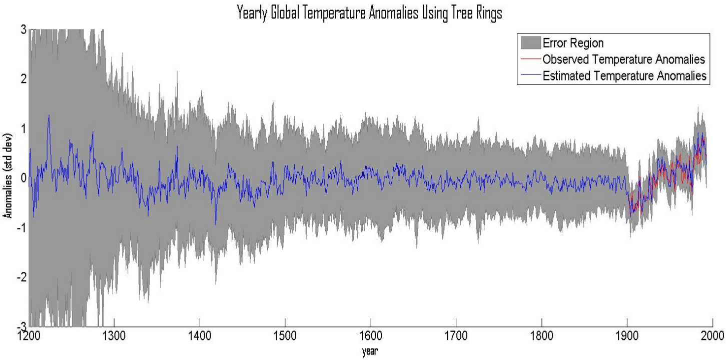
So

$$\|A - A_2\| \leq \|U - U_2\| \|S\| \|V^T\| \leq \|(G^T G)^{-1}\| \|G^T\| \|E_p\| S(1,1)$$

Using the formula for standard error with $M =$ number of sites,

$$E_A \leq \frac{\|A - A_2\|}{M}$$

The results of the error calculations



error grows larger further back in time because we have less trees.

Adding CO_2

- It is reasonable to assume that temperature is not the only variable affecting tree growth
- Adding a component for CO_2 and a constant "noise" factor may result in a more accurate approximation
- Thus the new equation is

$$GU_1 + CH + b = P_1$$

where C is the matrix of CO_2 values and b is a matrix of constants.

Reconstructing back in time with CO_2

To reconstruct the new time series, we solve the equation

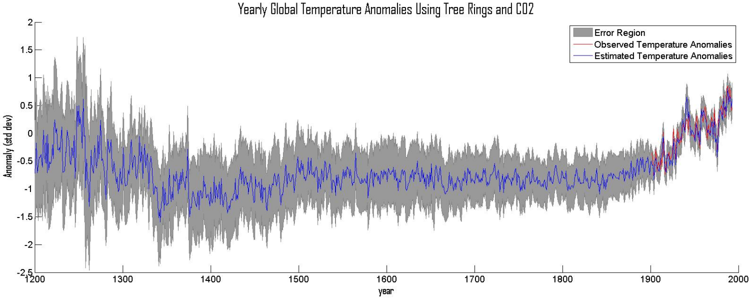
$$GU_2 = P_2 - CH - b$$

Since we know C, H, and b we can isolate U_2 on the lefthand side

Then we can reconstruct the temperature with the same equation as before:

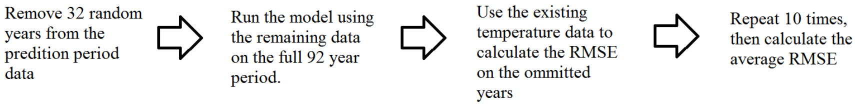
$$A_2 = U_2 S V^T$$

Temperature Anomaly Approximation with CO_2



Validating the Model

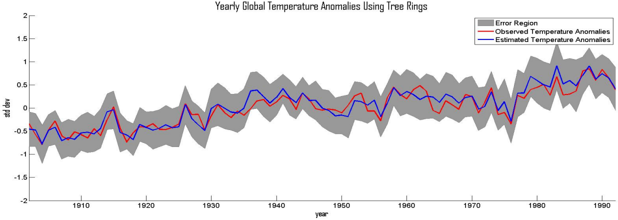
To see how the CO_2 affects the error, we follow this process of cross-validation:



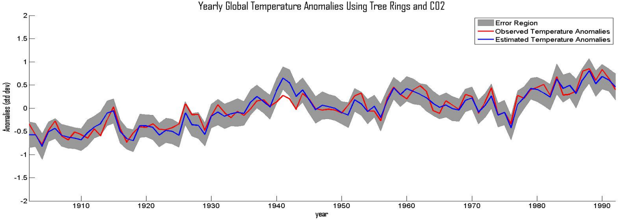
This process was followed with both models

Comparing the mean RMSE

No CO2: RMSE = 0.238



With CO2: RMSE =0 .177



Other Models to compare

- The "Berkeley Earth Averaging Process"
- Uses Kriging interpolation to create a smooth temperature field across land surface
- Only estimates during times with present historical records.
- Possible synthesis of this interpolation with our method?

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